4.4 Logarithmic and Exponential Equations MATH 1610 THOMPSON

1.) $log_5 x = 2$ $5^2 = x$ x = 25

2.) $log_3(11x) = 3$ $11x = 3^3$ $x = \frac{27}{11}$

3.) $log_8(x+6) = log_8 14$ **when bases equal they drop* x+6=14 x = 8

4.) $\frac{1}{3}log_2 x = 2log_2 3$ $\sqrt[3]{x} = 3^2$ $x = 9^3$ x = 729

5.) $5log_2 x = -log_2 32$ $x^5 = \frac{1}{32}$ $x = \sqrt[5]{\frac{1}{32}}$ $x = \frac{1}{2}$

exponent of (x-9)

6.) $2log_2(x-9) + log_2 2 = 5$ Condense $log_2 2(x-9)^2 = 5$ $\boxed{2^5 = 32}$

$2(x^2 - 18x + 81) = 32$ divide right by 2

$x^2 - 18x + 81 = 16$

$x^2 - 18x + 65 = 0$

$(x-5)(x-13)=0$ x = ~~5~~,**13** ***ONLY LARGER #***

7.) logx + log(x-15) = 2 *condense the logs (multiply since +)*

$log x(x-15) = 2$ *log with no base is **log₁₀***

$\log_{10}(x^2 - 15x) = 2$

$x^2 - 15x - 100 = 0$

$(x-20)(x+5) = 0$ x = ~~-5~~,**20** ***ONLY LARGER #***

8a.) $\log(4x+4) = 1 + \log(x-9)$ $\log(4x+4) - \log(x-9) = 1$

condense: $log_{10}\frac{4x+4}{x-9} = 1$ $\frac{4x+4}{x-9} = 10^1$ cross multiply

10x-90=4x+4

$x = \frac{47}{3}$

8b.) $\log(6x+5) = 1 + \log(x-5)$

$\log(6x+5) - \log(x-5) = 1$

condense: $log_{10}\frac{6x+5}{x-5} = 1$ $\rightarrow$ $\frac{6x+5}{x-5}=10$ $\rightarrow$ $10(x-5)=6x+5$ $\rightarrow$ $10x-50=6x+5$

$4x = 55$ $x = \frac{55}{4}$

8c.) $\log(3x+6) = 1 + \log(x-7)$ $\rightarrow$ $\log(3x+6)-\log(x-7)=1$

condense: $log_{10}\frac{3x+6}{x-7} = 1$ $\rightarrow$ $\frac{3x+6}{x-7} = 10$ $\rightarrow$ $10(x-7) = 3x+6$

$10x - 70 = 3x + 6$ $x = \frac{76}{7}$

9.) $log_3(x+10) + log_3(x+4) = 3$ $log_3(x+10)(x+4) = 3$

$(x+10)(x+4) = 3^3$

$x^2 + 14x + 40 = 27$

$x^2 + 14x + 13 = 0$

$(x+13)(x+1)=0$ $x = $ **-1**, $\cancel{-13}$ ($-1+11$) is positive

10.) $\log_2(x-5) = 3 - \log_2(x-3)$ $\rightarrow$ $\log_2(x-5)+\log_2(x-3)=3$

$log_2(x-5)(x-3) = 3$

$x^2 - 8x + 15 = 8$

$x^2 - 8x + 7 = 0$

$(x-7)(x-1)$ $x=7,1$ *can't have negative* $x = 7$

11.) $\log_{1/7}(x^2+x) - \log_{1/7}(x^2-x) = -1$ $\rightarrow$ $log_{1/7}\frac{x^2+x}{x^2-x} = -1$ $\rightarrow$

$\frac{x(x+1)}{x(x-1)} = \frac{1}{7}^{-1} = \frac{\cancel{x}(x+1)}{\cancel{x}(x-1)} = 7$

cross multiply after cancelling xs $7x-7 = x+1$

$6x = 8$ $x = \frac{4}{3}$

12a.) $\log_a(x - 2) - \log_a(x + 7) = \log_a(x - 3) - \log_a(x + 4)$

condense: $\cancel{\log_a} \frac{(x-2)}{(x+7)} = \cancel{\log_a} \frac{(x-3)}{(x+4)}$

$\frac{(x-2)}{(x+7)} = \frac{(x-3)}{(x+4)}$ CROSS MULTIPLY

$(x-2)(x+4) = (x+7)(x-3)$

$x^2 + 2x - 8 = x^2 - 4x - 21$

$-2x = -13$ $x = \frac{13}{2}$

12b.) $\log_a(x - 1) - \log_a(x + 2) = \log_a(x - 4) - \log_a(x + 2)$

condense: $\log_a \frac{(x-1)}{(x+2)} = \log_a \frac{(x-4)}{(x+2)}$ $\rightarrow \frac{(x-1)}{(x+2)} = \frac{(x-4)}{(x+2)}$

$(x-1)(x+2) = (x+2)(x-4)$

$x^2 + x - 2 = x^2 - 2x - 8$

$3x = -6$

$x = -2$ *** *log can't be negative*

NO SOLUTION

13.) $(\log_2 x)^2 - 5(\log_2 x) = 6$

Use u substitution: $u = \log_2 x$ $u^2 - 5u - 6 = 0$

$(x-6)(x+1) = 0$

$x = 6, -1$ THEN

$\log_2 x = 6$ $\log_2 x = -1$

$2^6 = 64$ $2^{-1} = \frac{1}{2}$

14.) $7^{x-3} = 49 \rightarrow$ same base on both sides

$\cancel{7}^{x-3} = \cancel{7}^2$ $x-3 = 2$

$x = 5$

15.) $8^x = 13 \rightarrow$ take **ln** of both sides

$ln8^x = ln13$ take ln of both sides

x = $\boxed{\frac{ln13}{ln8}}$ = 1.233 **round to 3 decimal places*

16.) $4^{-x} = 3.6 \rightarrow$ $\boxed{-log_4 3.6}$

x ≈ -0.924 **round to 3 decimal places*

17.) $5(3^{9x}) = 8 \rightarrow$

$3^{9x} = \frac{8}{5}$ *change to log form* $\log_3 (\frac{8}{5})=9x$ *change base formula*

$9x = \frac{\ln(\frac{8}{5})}{ln3} \rightarrow x = \boxed{\frac{\ln(\frac{8}{5})}{(9\ln(3))}} \approx 0.048$ **use parentheses in calculator*

divide by 9 – move it to the bottom

18.) $2^{1-5x} = 5^x \rightarrow$ TAKE *ln* OF BOTH SIDE

$=ln2^{1-5x} = ln5^x$ *exponent moves out in front then distribute*

(1-5x)ln2 = xln5

ln2 = xln5+5xln2 *move x to the right to keep it positive*

ln2= x ln5 +5xln2

ln2=x(ln5+5ln2) = *factor out the x*

x = $\boxed{\frac{ln2}{ln5+5ln2}}$ x≈ 0.137

19.) $\pi^{1-5x} = e^{2x} \rightarrow$ TAKE *ln* OF BOTH SIDE

$= ln\pi^{1-5x} = \cancel{e}^{2x}$ *exponent moves out in front then distribute*

$(1-5x)\ln\pi = 2x$

$\ln\pi - 5x\ln\pi = 2x$ *move x to the right to keep it positive*

$\ln\pi = 5x\ln\pi + 2x$

$\ln\pi = x(5\ln\pi+2x)$ *factor out the x*

$x = \boxed{\frac{ln\pi}{5ln\pi+2}}$ $x \approx 0.148$

20.) $2^{2x} + 2^x - 56=0 \rightarrow$ let $u = 2^x$

You can always use ln..... $\frac{ln7}{ln2} = 2.807$

$u^2 + u - 56 = 0$

$(u + 8)(u - 7) = 0$ $u = -8,7$

$2^x = -8$
no solution

$2^x = 7$

$x = \boxed{\log_2 7}$ *base on bottom*

$x = \frac{log7}{log2} \approx \mathbf{2.807}$

21.) $3^{2x} + 3^{x+1} - 54=0 \rightarrow$ let $u = 3^x$

You can always use ln..... $\frac{ln6}{ln3} = 1.631$

$u^2 + 3u - 54 = 0$

$(u + 9)(u - 6) = 0$ $u = -9,6$

$3^x = -9$
no solution

$3^x = 6$

$x = \boxed{\log_3 6}$ *base on bottom*

$x = \frac{log6}{log3} \approx \mathbf{1.631}$

22.) $4^x + 2^{x+1} - 15=0 \rightarrow$ let $u = 2^x$

You can always use ln..... $\frac{ln3}{ln2} = 1.585$

$u^2 + 2u - 15 = 0$

$(u + 5)(u - 3) = 0$ $u = -5,3$

$2^x = -5$
no solution

$2^x = 3$

$x = \boxed{\log_2 3}$ *base on bottom*

$x = \frac{log3}{log2} \approx \mathbf{1.585}$

23.) $36^x - 8 \cdot 6^x = -16$ $\rightarrow$ let $u = 6^x$

You can always use ln..... $\frac{\ln 4}{\ln 6} = 0.774$

$u^2 - 8u - 16 = 0$

$(u - 4)(u - 4) = 0$ $u = 4$ $6^x = 4$

$x = \boxed{\log_6 4}$ *base on bottom*

$\mathbf{x = \frac{\log 4}{\log 6} \approx 0.774}$

24.) $4^x - 10 \cdot 4^{-x} = 3$ multiply all by 4^x **cancels the middle one*

$(4^x)^2 - 10 \cdot 1 = 3 \cdot 4^x$ $\rightarrow$ let $u = 4^x$

You can always use ln..... $\frac{\ln 5}{\ln 4} = 1.161$

$u^2 - 3u - 10 = 0$

$(u - 5)(u + 2) = 0$ $u = -2$ $4^x = 5$

$x = \boxed{\log_4 5}$ *base on bottom*

$\mathbf{x = \frac{\log 5}{\log 4} \approx 1.161}$

25.) $(\sqrt[5]{7})^{3-2x} = 7^{x^2}$ **get the base the same to cancel*

$7^{\frac{1}{5}(3-2x)} = 7^{x^2}$ **set exponents equal and multiply the right side by 5*

$3 - 2x = 5x^2$

$5x^2 + 2x - 3 = 0$ *slide and divide* $x^2 + 2x - 15 = 0$

$(x+5)(x-3) = 0$ $x = -1, \frac{3}{5}$

26.) The population of a certain country in 1998 was 281 million people. In addition, the population of the country was growing at a rate of 0.8% per year. Assuming that this growth rate continues, the model $P(t) = 281(1.008)^{t-1998}$ represents the population P (in millions of people) in year t.
According to this model, when will the population of the country reach 391 million people?

$391 = 281(1.008)^{t-1998}$

$\frac{391}{281} = (1.008)^{t-1998}$ *take ln of both sides

$\ln\left(\frac{391}{281}\right) = (t-1998)\ln 1.008$ *divide by (ln1.008) first then add 1998

$\frac{\left(\ln\left(\frac{391}{281}\right)\right)}{\ln 1.008} + 1998 = t$ t = 2039

Ex.) $\ln x + \ln(x+2) = 4$ $\ln x + \ln(x+2) = 4$

$\ln(x)(x+2) = 4$

$x^2 + 2x = e^4$

$x^2 + 2x - e^4 = 0$

$\frac{-2 \pm \sqrt{4 - 4(1)(-e^4)}}{2} =$ *factor out the 4 under radical*

$\frac{-2 \pm \sqrt{4(1+e^4)}}{2}$ *take square root of the 4*

$\frac{-2 \pm 2\sqrt{1+e^4}}{2} = -1 \pm \sqrt{1+e^4}$

Ex.) $\ln x + \ln(x+4) = 4$ $\ln x + \ln(x+4) = 4$

$\ln(x)(x+4) = 4$

$x^2 + 4x = e^4$

$x^2 + 4x - e^4 = 0$

$\frac{-4 \pm \sqrt{16 - 4(1)(-e^4)}}{2} =$ *factor out the 4 under radical*

$\frac{-4 \pm \sqrt{4(4+e^4)}}{2}$ *take square root of the 4*

$\frac{-4 \pm 2\sqrt{4+e^4}}{2} = -2 \pm \sqrt{4+e^4}$